Heuristic quantization of the cranking model

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The cranking model, which is able to describe nonlinear effects of nuclear rotation from a microscopic view point, has been extensively applied to studies of rapidly rotating nuclei. However, the semi-classical treatment of the angular momentum prevents the model from being directly applied to the study of the electromagnetic transition probabilities at low spin. The unified model has been a basic tool to analyze the electromagnetic transition rates of rotational bands. In this model, although the nuclear rotation is described by macroscopic ("rotor") wave functions, the quantum-mechanical angular-momentum algebra is precisely taken into account. Thus, these two models, the cranking and unified models, have complementary merits and demerits. If we can combine the merits of both models, we may be able to obtain a feasible method of microscopically calculating transition probabilities of rotational bands. In this paper, we introduce a simple "quantization" procedure for the cranking model and discuss Coriolis coupling effects on the $I^\pi = 3^-$ octupole states as an example of its application. See Ref. [1] for details.

Since the *quantum* angular momentum becomes approximately *classical* in the high-spin limit, the transition probabilities can be estimated at this limit. Marshalek derived a simple formula for the reduced transition amplitude in the leading order of $1/I$ [2];

$$\frac{\langle I_{\rm f}||Q_\lambda||I_{\rm i}\rangle}{\sqrt{2I_{\rm i}+1}} \approx \langle {\rm f}|\widetilde{Q}_{\lambda\mu=\Delta I}|{\rm i}\rangle \ , \tag{1}$$

where $\Delta I = I_{\rm f} - I_{\rm i}$ and $\widetilde{Q}_{\lambda\mu}$ is the transition operator defined with respect to the cranking (rotation) axis, $\widetilde{Q}_{\lambda\mu} = i^{-\mu}\sum_\nu d^\lambda_{\mu,\nu}\,(-\pi/2)\,Q_{\lambda\nu}$ (the operator defined with respect to the symmetry axis is denoted as $Q_{\lambda\nu}$ without a tilde). The initial and final states in the cranking model, $|{\rm i}\rangle$ and $|{\rm f}\rangle$, have good signature-quantum numbers and are constructed as a linear combination of $|\pm K_{\rm i(f)}\rangle\!\rangle$ which have good K-quantum numbers in the limit of $\omega_{\rm rot} = 0$. One may understand that the direction of the angular momentum exactly coincides with the cranking axis in this formula, which is valid only in the high-spin limit.

If we simply apply the formula (1) to the low-spin limit ($\omega_{\rm rot} = 0$), we obtain for the K-allowed transitions within the first order with respect to $\omega_{\rm rot}$,

$$\frac{\langle I_{\rm f}||Q_\lambda||I_{\rm i}\rangle}{\sqrt{2I_{\rm i}+1}} = C_{\rm if}\left(\Big[\langle\!\langle K_{\rm f}|Q_{\lambda\Delta K}|K_{\rm i}\rangle\!\rangle\Big]_0 d^\lambda_{\Delta I,\Delta K}\Big(-\frac{\pi}{2}\Big) + \sum_{\rho=\pm1}\Big[\frac{d\langle\!\langle K_{\rm f}|Q_{\lambda,\Delta K+\rho}|K_{\rm i}\rangle\!\rangle}{d\omega_{\rm rot}}\Big]_0\omega_{\rm rot}\, d^\lambda_{\Delta I,\Delta K+\rho}\Big(-\frac{\pi}{2}\Big)\right) \ , \tag{2}$$

where $\Delta K = K_{\rm f} - K_{\rm i}$ and the overall factor $i^{-\Delta I}$ and the signature-dependent terms like $\langle\!\langle K_{\rm f}|Q_{\lambda\nu}|-K_{\rm i}\rangle\!\rangle$ were omitted in the r.h.s. for simplicity. $C_{\rm if} = 1$ if $K_{\rm i} = K_{\rm f} = 0$ or $K_{\rm i} \neq 0$, $K_{\rm f} \neq 0$, otherwise $C_{\rm if} = \sqrt{2}$. Here $[*]_0$ means that the expression is evaluated by taking the limit $\omega_{\rm rot} \to 0$. Since the d function $d^\lambda_{\Delta I,\Delta K}(-\pi/2)$ accounts for only the classical geometry of angular momentum (the argument $-\pi/2$ means the direction of the angular momentum vector is perpendicular to the symmetry axis), Eq.(2) cannot describe the correct intensity relations at low spin. We need to quantize this equation.

In order to quantize the semi-classical angular momentum, again we consider the high-spin limit where quantum effects become less important. In the limit of $I_{\rm i}, I_{\rm f} \gg K_{\rm i}, K_{\rm f}$,

$$d^\lambda_{\Delta I,\Delta K}(-\frac{\pi}{2}) \approx \langle I_{\rm i}K_{\rm i}\lambda\Delta K|I_{\rm f}K_{\rm f}\rangle \ , \qquad \text{with} \quad \Delta I = I_{\rm f} - I_{\rm i} \text{ and } \Delta K = K_{\rm f} - K_{\rm i} \ . \tag{3}$$

Thus, if we replace the d function in the leading-order term of Eq.(2) by the equivalent Clebsch-Gordan coefficient (3), we obtain the same structure as that of the leading-order intensity relation in the unified model, $C_{\rm if}\langle\!\langle K_{\rm f}|Q_{\lambda\Delta K}|K_{\rm i}\rangle\!\rangle\langle I_{\rm i}K_{\rm i}\lambda\Delta K|I_{\rm f}K_{\rm f}\rangle$. This replacement allows us to apply this formula at low spin because the C.-G. coefficient $\langle I_{\rm i}K_{\rm i}\lambda\Delta K|I_{\rm f}K_{\rm f}\rangle$ incorporates the quantum angular-momentum algebra.

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In order to generalize this replacement to higher-orders, it is convenient to rewrite Eq.(3) as

$$d^{\lambda}_{\Delta I,\Delta K}\Big(-\frac{\pi}{2}\Big) \;\rightarrow\; \frac{\langle K_{\rm f} I_{\rm f}||\mathcal{D}^{\lambda}_{*,\Delta K}||K_{\rm i} I_{\rm i}\rangle_{\rm unsym}}{\sqrt{2I_{\rm i}+1}} \;. \tag{4}$$

This means that the d function in Eq.(2) corresponds to the reduced matrix element of the $\mathcal{D}$ operator with respect to the rotor part of the unsymmetrized wave functions in the unified model. Roughly speaking, this may be seen as a "quantization"; d function $\rightarrow$ $\mathcal{D}$ operator. Using the relation $\omega_{\rm rot} \approx \langle J_x\rangle/\mathcal{J} \rightarrow I_{\pm}/\mathcal{J}$, now we may introduce a "quantization" rule for the first-order correction terms (the second terms in the r.h.s. of Eq.(2));

$$\omega_{\rm rot}\, d^{\lambda}_{\Delta I,\Delta K\pm 1}\Big(-\frac{\pi}{2}\Big) \;\rightarrow\; \frac{1}{\mathcal{J}}\frac{\langle K_{\rm f} I_{\rm f}||\,\frac{1}{2}\{I_{\pm},\mathcal{D}^{\lambda}_{*,\Delta K\pm 1}\}\,||K_{\rm i} I_{\rm i}\rangle_{\rm unsym}}{\sqrt{2I_{\rm i}+1}} \;, \tag{5}$$

where $\mathcal{J}$ is the moment of inertia of the band under consideration at $\omega_{\rm rot}=0$. The appropriate operator ordering in the r.h.s. is discussed in Ref. [1]. These higher-order terms take account of the Coriolis-coupling effects. It is worth noting that, with $\omega_{\rm rot} = I/\mathcal{J}$, the the l.h.s. and r.h.s. of Eqs.(4, 5) become exactly equivalent in the high-spin limit, because the *quantum* values (r.h.s.) are approaching the *classical* values (l.h.s.) at this limit.

Using these "quantization" rules, one can obtain the explicit intensity formula for the K-allowed transitions including the lowest-order corrections from Coriolis coupling. In Ref. [1], we also discuss the quantization for the K-forbidden transitions and show explicit formulae for several applications to the intra- and inter-band transitions.

The major achievement of this "quantization" is to provide a feasible method of microscopically calculating the intrinsic parameters of rotational intensity relations including the effects of Coriolis coupling. As an example of the applications, we discuss here the Coriolis coupling effects for $I^{\pi}=3^{-}$ octupole states. Neergård and Vogel [3] have shown that Coriolis mixing among the octupole states is important even at low spins. The relative differences of B($E3$; $3^{-}\rightarrow 0^{+}$) values among the different K modes cannot be understood without Coriolis-coupling effects. Fig.1 shows the results of the cranked RPA calculation for the $E3$ amplitudes associated with the octupole-vibrational bands ($K=0\sim 3$) in Gd isotopes. The Coriolis coupling concentrates the $E3$ strengths onto the lowest-lying octupole state ($K=1$ in ^{156,158}Gd and $K=2$ in ^{160}Gd). The large differences of B($E3$) values between the $K=0$ and 1 bands observed in ^{156,158}Gd disagrees with the leading-order calculation without the Coriolis coupling (open circles), while it is reproduced in the calculation with the lowest-order correction terms (solid circles). This example shows the usefulness of the new quantization method.

References

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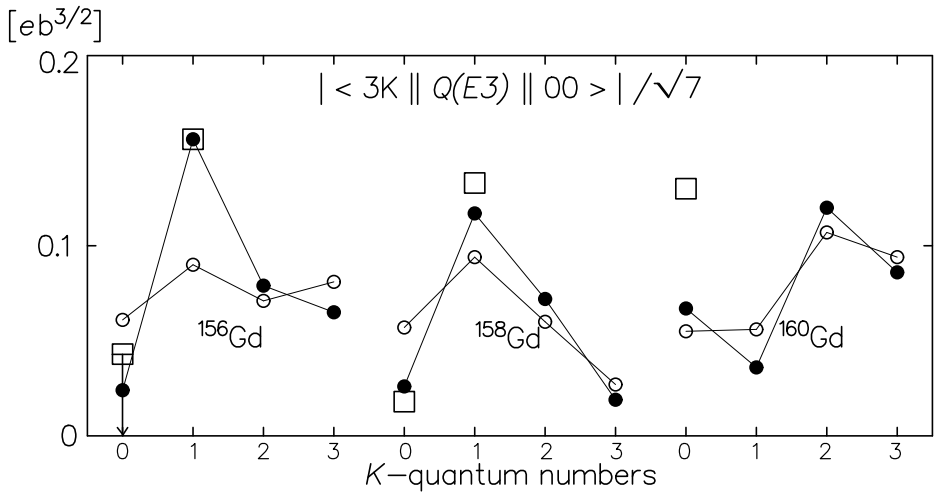

Fig.1 : The $E3$ amplitudes for Gd isotopes calculated by means of the RPA based on the cranked shell model. Solid (open) circles indicate the results with (without) the lowest-order Coriolis coupling terms. Experimental data are denoted by squares. See Ref. [1] for details of the calculations.